

# ST JOHN'S COLLEGE, UNIVERSITY OF OXFORD

## The Deep Mathematical Theory of Selfish Genes

### FURTHER PARTICULARS

The College intends to appoint two two-year Fixed-term Research Associates to work in its Research Centre. The college will host a further two years of the project and provide matching funding, if external funding can be obtained. The successful applicants will work with Professors Alan Grafen and Charles Batty on an abstract mathematical project that underpins and develops modern Darwinian theory.

#### About the project

Alan Grafen has been publishing papers on 'formal Darwinism' since 1999. The quickest way to appreciate the project to date is to consult

- 1) a simple verbal account of the work to date (A. Grafen 2007. The formal Darwinism project: a mid-term report. *Journal of evolutionary Biology* 20, 1243-1254. doi:10.1111/j.1420-9101.2007.01321.x), available at <http://users.ox.ac.uk/~grafen/cv/fdpmidterm.pdf>
- 2) an elementary mathematical introduction (A. Grafen 2008. The simplest formal argument for fitness optimisation. *Journal of Genetics*, 87, 421-433. doi: 10.1007/s12041-008-0064-9 ) available at <http://users.ox.ac.uk/~grafen/cv/simplest.pdf>
- 3) a reasonably complete form of the argument but using finite mathematics for 'inclusive fitness' (A Grafen 2006 Optimization of inclusive fitness. *Journal of theoretical Biology* 238, 541-563), available at <http://users.ox.ac.uk/~grafen/cv/optif.pdf>, and
- 4) a paper using the heavier mathematical tools likely to be required for future work, applied to the problem of allowing the existence of different classes of individual (A. Grafen 2006. A theory of Fisher's reproductive value. *Journal of mathematical Biology* 534, 15-60.), available at <http://users.ox.ac.uk/~grafen/cv/classes.pdf>

The work now requires collaboration with mathematicians, in order to tackle more abstract problems, to solve them more efficiently, and to publish the results in a mathematically acceptable way. This has brought about the involvement of co-applicant Charles Batty, a pure mathematician, and this advertisement for two research associates.

The postholders will be involved in linking the mathematics of motion with the mathematics of optimisation at an abstract level. The tools will include measure theory and Markov processes over compact Hausdorff spaces. The goals of the project are to unify, formalise and extend existing results into a rigorous and general statement about the connections between natural selection, on the one hand, and optimality in the design of organisms, including a precise definition of 'optimal', on the other.

The rest of this section gives some background as a shorter alternative to reading the four papers given above. The concept of fitness optimization is routinely used by field biologists, and first-year biology undergraduates are frequently taught that natural selection leads to organisms that maximize their fitness. Dawkins' *The Selfish Gene* (1976) promoted a conceptual integration of modern evolutionary theory in which genes are viewed as optimising agents, which is extremely influential and widespread today and encompasses inclusive fitness theory and evolutionarily stable strategies as well as general optimality ideas. However, mathematical population geneticists mainly deny that natural selection leads to optimization of any useful kind. This fifty-year old schism is intellectually damaging in itself, and has prevented improvements in our concept of what fitness is. One underlying cause is that the link between natural selection and fitness optimization is much more sophisticated than the usual optimization principles associated with dynamical systems, namely Lyapunov functions and gradient functions.

The aim is to formalize relevant links between the mathematics of motion (representing the known process of gene frequency change as the dominant mechanism of evolution) and the mathematics of optimization, in a rigorous way. Generality is important, as a major aim is to find mathematical arguments that match Darwin's verbal arguments in the *Origin of Species*, as well as Dawkins's verbal arguments in the *Selfish Gene* and later works. This requires, for example, the use of measure theory to represent populations and to represent uncertainty. The aim is to model fully the core of the underlying verbal arguments, so that we can be confident there is 'nothing left in them'.

The construction of such a framework will have major implications for the conceptual basis of biology. For example, the understanding of organisms as agents is given a rigorous meaning, which is of interest to philosophers. The definition of fitness, i.e. the quantity that organisms are assumed to maximize, is often controversial in biology, and this project will settle many, at least, of those questions. To take one simple controversial example, when there is uncertainty in fitness, will natural selection lead to organisms that maximize the geometric, or the arithmetic, mean over that uncertainty? Thus this highly abstract mathematical project will have significant implications at many different levels in biology. It will also be of interest to historians of science, as it will claim to show the underlying logic of Darwin's great insight and of Dawkins' conceptual unification.

For those seeking more information on the work to be done, excerpts from the grant application are provided at the end of this document.

### **Intellectual environment**

The postholders will be encouraged to participate in the intellectual life of the college and of the Research Centre, as well as that of the Mathematical Institute and the Zoology Department, both of which are nearby. There is active interest in Zoology in the formal Darwinism project and its development (Prof. Stu West, Dr Ashleigh Griffin, Dr Andy

Gardner, and their groups). Prof. Philip Maini, Director of the Centre for Mathematical Biology, is a fellow of St John's.

### **Principal responsibilities**

1. To carry forward the program of generalization, unification and extension of formal Darwinism set out in the grant application, under the direction of, and in collaboration with, Alan Grafen and Charles Batty. Extracts from the grant application are attached as an appendix to these further particulars.
2. To prove theorems embodying the results.
3. To write papers for submission to mathematical academic journals presenting these theorems.
4. To attend conferences to engage with other mathematical biologists, learn from their work, and to present the results of the project.
5. To work with Alan Grafen and Charles Batty in finding the extra two years of matching funding to extend the project from two to four years.

### **Selection criteria**

Candidates must have completed, or be about to complete, a doctorate in a suitable branch of Mathematics, or in another subject but with a strong pure mathematical component. Candidates should be high-quality rigorous mathematicians, and comfortable with some of measure theory, probability theory, stochastic processes and optimization. Their doctorates need not be in any of those areas, but the candidates should have a working knowledge of some of them and be prepared to be careful on details. An interest in the biological implications of the work would be encouraging but is not necessary. The successful candidates will be expected to contribute to the project's intellectual development and must be able to work independently.

### **The College and the Research Centre**

St. John's College is one of the larger colleges in the University of Oxford. Conveniently situated on St. Giles, it was founded in 1555 by Sir Thomas White and has around 600 students and 80 Fellows. The St John's College Research Centre was formally set up by the Governing Body of St John's College on February 14th 2001. Its aim is to provide focus and support for the College's intellectual and academic life as it already exists and to support new research, particularly of an interdisciplinary nature which might otherwise be unfunded: and to enhance the College's role in promoting first-class innovative research in the University of Oxford and the academic community at large. It supports a balanced mixture of research in science and humanities. It supports longer programmes of two years of full support, with two further years of matched-funding support, as well as a variety of shorter projects. For further information about the College and the

Research Centre, please visit our web-sites at <http://www.sjc.ox.ac.uk> and <http://www.sjc.ox.ac.uk/386/Research-Centre.html>.

## **Terms and Conditions of Employment**

**Salary:** Appointment will be on the University's Grade 7 for Academic and Academic-related staff, currently starting at £29,099 per annum, with annual increments.

**Holidays:** Annual leave entitlement is 30 days, which includes 5 fixed days to be taken at Christmas, plus 8 public holidays. The holiday year begins on 1 January and ends on 31 December. Leave must be approved in advance by Prof. Grafen.

**Meals:** The postholder will be entitled to lunches and dinners in the Senior Common Room for the forty-eight or so weeks each year the kitchen is open.

**Pension:** The postholder will be entitled to join the Universities' Superannuation Scheme.

**Probationary and notice periods:** The appointment is subject to satisfactory completion of a six-month probationary period, during which the notice period will be one month on either side. Once the appointment has been confirmed, the notice period on either side will be three months.

## **Appointment procedure**

Applications (original plus four copies) consisting of a covering letter, curriculum vitae, list of publications and a succinct (1 - 3 page) statement outlining research interests and experience should be sent to the Academic Administrator, St John's College, Oxford, OX1 3JP. Please remember that you have the option to include a completed recruitment monitoring form with your application, and doing so will improve the college's ability to monitor its procedures.

The application should be received by the College no later than 4 May 2011. E-mailed and faxed applications will not be accepted. Interviews are likely to be held by the end of May.

Please give the names, addresses and telephone numbers of three referees. The College will assume that it is free to approach referees at any stage unless the candidate's application stipulates otherwise. Candidates who wish a referee or referees to be approached only with their specific permission, and/or if they are shortlisted for interview, are asked to state such requirements alongside the details of the relevant referee(s).

Applicants who would need a work visa if appointed to the post are asked to note that under the UK's points-based migration system they will need to demonstrate that they have sufficient points, and in particular that: (i) they have sufficient English language

skills (evidenced by having passed a test in basic English, *or* coming from a majority English-speaking country, *or* having taken a degree taught in English) *and* (ii) that they have sufficient funds to maintain themselves and any dependants until they receive their first salary payment.

Further information is available at:

<http://www.ukba.homeoffice.gov.uk/workingintheuk/tier2/generalarrangements/eligibility/>.

### **Data protection**

All data supplied by applicants will be used only for the purposes of determining their suitability for the post, and will be held in accordance with the principles of the Data Protection Act 1998 and the College's Data Protection Policy.

### **Equality of Opportunity**

St John's College subscribes to the University of Oxford's policy statement concerning equality of opportunity which states that 'The policy and practice of the University of Oxford require that all staff are afforded equal opportunities within employment and that entry into employment with the University and progression within employment will be determined only by personal merit and the application of criteria which are related to the duties of each particular post and the relevant salary structure. In all cases, ability to perform the job will be the primary consideration. Subject to statutory provisions, no applicant or member of staff will be treated less favourably than another because of his or her sex, marital status, sexual orientation, racial group, age or disability.'

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**ST JOHN'S COLLEGE RECRUITMENT MONITORING**

The policy and practice of the College require that entry into employment with the College and progression within employment will be determined only by personal merit and the application of criteria which are related to the duties of each particular post. Subject to statutory provisions, no applicant or member of staff will be treated less favourably than another because of his or her gender, marital or civil partnership status, sexual orientation, religion or belief, racial group, age or disability. In all cases, ability to perform the job will be the primary consideration. Completion of this form is voluntary. If you choose to do so, please return the form to the **Academic Administrator, St John's College, OX1 3JP**.

Any information given will be used only to support the College's equal opportunities policy and in accordance with the principles of the Data Protection Act 1998. It is not part of the selection process and will not be seen by any person interviewing you for employment. The information you give will be retained only for statistical purposes and will not be linked to your name. But NB if the appointee to the post is a migrant sponsored under the UK's new points-based migration system, we are required to retain all applications for the duration of the sponsorship.

*Please answer the questions by ticking as appropriate.*

1. **Are you**            Female            Male

2. **Do you have a disability?** If yes, please answer question 3.            Yes            No

3. **Please tick one or more boxes to describe your disability**

- |                            |                                   |
|----------------------------|-----------------------------------|
| Blind/partially sighted    | Dyslexia                          |
| Deaf/hard of hearing       | Mental Health difficulties        |
| Wheelchair user            | Unseen disability (e.g. diabetes) |
| Other mobility impairments | Other (please describe).....      |

4. **Nationality** Please tick one box only.

UK            Other EC            Other (please specify).....

5. **How long have you lived in the UK?** Please tick one box only

I do not currently live in the UK            I have always lived in the UK

<1 year 1-4 years 5-10 years 11-20 years 21+years

6. **Age** Please tick one box only

18-25 26-30 31-35 36-40 41-45 46-50 51-55 56+

7. **Please tick one box only to describe your ethnic origin**

(Ethnic origin questions are not concerned with nationality, place of birth, or citizenship. They are concerned with colour and ethnic group. Citizens of any country may belong to any of the groups indicated. The ethnic categories used here are based on the 2001 census of the population and are those required by the Higher Education Statistics Agency.)

<b>White</b>	<b>Black or Black British</b>	<b>Chinese</b>
British	Caribbean	Chinese
Irish	African	
Any other White background	Any other Black background	

Please specify.....Please specify.....

<b>Mixed</b>	<b>Asian or Asian British</b>	<b>Other ethnic group</b>
White and Black Caribbean	Indian	Any other
White and Black African	Pakistani	Please specify.....
White and Asian	Bangladeshi	
Any other mixed background	Any other Asian background	

Please specify..... Please specify.....

## Extracts from the grant application to the St John's Research Centre

### **Objectives**

1. To construct a mathematical framework, with appropriate theorems, to represent fully the core argument in Darwin's *Origin of Species*, namely that the purely mechanical processes of inheritance and reproduction can give rise through natural selection to the appearance of design.
2. To include in this framework the significant later additions to the theory of natural selection, namely sexual selection, inclusive fitness and the concept of evolutionarily stable strategies.
3. To extend the formal links between equations of motion and the mathematics of optimisation begun by Grafen (1999, 2002, 2006a, 2006b), as a formal representation of what biologists have meant by 'fitness optimisation', and so clear up a misunderstanding with mathematical population geneticists.
4. To construct the framework as generally as possible, in order to fulfil objectives 1 to 3 as generally as possible. For example allowing arbitrary population size (finite or infinite) and arbitrary uncertainty (finite or infinite set of states of nature, with arbitrary distribution of probability), as well as arbitrary genetic architecture. Social behaviour and game-like interactions should also be permitted, as should a very general generational structure that permits overlapping generations and does not specify whether time is continuous or discrete.

### **Summary**

Grand theories in physics are usually expressed in mathematics. Newton's mechanics and Einstein's theory of special relativity are essentially equations. Words are needed only to interpret the terms. Darwin's theory of evolution by natural selection has obstinately remained in words since 1859. Of course, there are many mathematical models that show natural selection at work, but they are all examples. None claims to capture Darwin's central argument in its entirety. In its grandest conception, this project aims to do just that, and even to include all the valid additions to the theory, namely sexual selection, the merging of Darwinism and Mendelism, inclusive fitness and evolutionary game theory.

The core argument is that the mechanical processes of inheritance and reproduction, today represented by equations of motion of gene frequency change, can give rise through natural selection to the appearance of design, today represented by the mathematics of optimisation. The formal approach is therefore to construct links between equations of motion and optimisation programs. The project has been substantially begun, and most of the ingredients identified. But the main work requires to be done of constructing a single overarching model of the optimising tendency of natural selection, and that is the work proposed in this project.

Such a formal and systematising effort will have many advantages. It will resolve a persistent and damaging forty-year-old misunderstanding within biology about what fitness optimisation means. Indeed it will greatly assist in resolving persistent doubts about what fitness itself means. The effort will also iron out various wrinkles in evolutionary theory, including the question of whether the arithmetic or geometric mean of fitness is maximised in the presence of uncertainty. A further advantage is that by setting up a formal version of Darwin's argument, it will raise the bar for those who claim to have their own understanding of Darwin: in the first place, it will need to be checked against the formal version; if it fails that test, it will still of course be possible to argue that the current formal version is incomplete or erroneous; *but* this will be a technical exercise that requires real intellectual work to be persuasive, for which words alone will not suffice.

Many biologists rely on fitness optimisation for giving meaning to their research questions, even though until now the concept has been somewhat dubious in mathematical terms. The project aims to provide a justification for this reliance, subject to making precise what fitness is.

This project is in many ways a mathematical, formal version of the argument of *The Selfish Gene*. There, Dawkins articulates in words a unifying structure for all the central adaptive theories used by evolutionary biologists, and grounds that unifying structure in a fully logical framework. A mathematical version will provide more precision, and answer a class of objections.

### **Detailed Application**

#### Background: the formal Darwinism project

The idea that organisms maximise their fitness as a result of natural selection is extremely important in many areas of biology. The explanatory apparatus of most whole organism, behavioural ecology, work would make no sense without it. However, the logical basis for

the idea is in considerable doubt. The mainstream of mathematical population geneticists since about 1964 has emphatically rejected the claim that fitness is maximised. Ewens (2004) discusses an optimisation principle that is a generalisation of a Lyapunov function, but that optimisation is an analogy to help understand the dynamical system and where it will go next, and not about whether individuals in the population are maximising their fitness subject to physical, physiological and informational constraints. The early denial of optimisation has matured into a later acceptance, but of a kind of optimisation that is quite different from the one needed in behavioural ecology. There has been essentially no formal consideration of the kind of optimisation that emerges so naturally from verbal arguments such as those of Darwin (1859) and Dawkins (1976).

The 'formal Darwinism project' aims to reconcile the mathematical and empirical approaches, by finding a mathematically defensible optimisation principle, which is very different from those considered and rejected in the past. A high-level description is that the project provides a mathematical, rigorous link between equations of motion (difference or differential equations representing gene frequency trajectories) and optimisation programs (the formal mathematical way of representing design and optimisation, expressing in a precise way the sense in which biologists believe that organisms maximise their 'fitness'); or more concisely, between population genetics and game theory. Only in the process of proving links between the mathematics of motion and of optimisation can the precise nature of 'fitness' emerge.

Obstacles in the project emerge from the requirement that the framework should capture the whole of Darwin's argument, and not represent merely a special case. (1) Population size. Darwin did not consider whether his populations were finite or infinite and, although all real populations are finite, many models have infinite populations. To embrace all the possibilities at once, the population must be modelled as a measure space. (2) Uncertainty. Chance effects are ubiquitous, and often modelled in biology. The set of possible outcomes is sometimes modelled as finite (e.g. hot or cold, or wet or dry) and sometimes as infinite with a special structure (e.g. a temperature belongs to the real numbers). To allow for arbitrary uncertainty, the outcomes must also be modelled as a measure space. (3) Genetic architecture. It is possible to make a great deal of progress without assuming very much about genetic architecture, and the Price Equation (Price 1970) is very permissive. Most other existing models assume a fixed (usually small) number of loci with fixed numbers of alleles at each generation, while Quantitative Genetics assumes very many loci are involved. (4) Generational structure. Most models assume discrete non-overlapping generations, some have continuous generations, and a few have discrete overlapping generations. Darwin didn't consider this question, and the force of his arguments doesn't seem to depend on this aspect – to capture his position, our mathematical framework must be as relaxed as possible about generational structure.

Grafen has so far published five papers in the project. In 1999, he set out the scope of the project, and showed how bet-hedging could be illuminated. In 2000, he developed the Price Equation with uncertainty for multigenerational models, answering concerns about dynamic insufficiency. Then in 2002 he published the first formal link between the Price equation and an optimisation program, a prototype of the very general model that is the goal of the project. The 2002 link included arbitrary uncertainty and arbitrary genetic architecture and arbitrary population size, but was limited in scope as follows: discrete non-overlapping generations were assumed, all individuals were assumed to be the same in an abstract sense, and social behaviour was excluded. One key question was what kinds of results could actually be proved: four reasonably strong propositions establish links between population genetics and optimisation programs, but one strand of thought in the ongoing project must be whether stronger links are possible.

Two papers published in 2006 take things forward from 2002. (A sabbatical leave partly funded by a Leverhulme fellowship permitted Grafen to complete these papers.) In 2006a, he constructed a link that required finite population size and a finite number of outcomes of uncertainty, but permitted social actions, and developed a mathematically rigorous and explicit version of the inclusive fitness models of Hamilton (1964, 1970). In 2006b, he developed a link that did not include uncertainty, but did allow 'classes', so that individuals could differ in sex and/or weight and/or height and/or other factors.

Each of these steps provided new insights about evolution.

1. Uncertainty. The maximisation of the geometric mean of absolute fitness is equivalent to the maximisation of the arithmetic mean of relative fitness (to the population mean). This allows bet-hedging phenomena to be incorporated into a framework in which the arithmetic mean of fitness is maximised. (1999 paper)
2. Symmetry of individuals. The population in the population genetic model corresponds to a single implicit decision-taker in the optimisation program. A latent assumption in optimisation thinking, of some kind of symmetry between individuals, is brought out by the need to forge explicit links. The details vary with the case, but are biologically significant. (2002 and 2006a papers)
3. Inclusive fitness. The arithmetic mean of relative inclusive fitness is the maximand, but 'relative' is to the mean personal fitness not to the mean of the absolute inclusive fitness. (2006a paper)
4. Reproductive value has to be defined for classes not present in equilibrium, and the apparatus for doing so depends on links between forwards and backwards processes. (2006b paper)

So far, Grafen has pursued this whole project by himself, but consulting Batty when necessary on an informal basis. It is now important for the work to be done in collaboration with mathematicians, and supervised by a mathematical PI. There are four reasons for this. (1) Time. In six month's leave, Grafen completed one significant paper and started and finished another – but while engaged in the usual academic round, progress is slow in this challenging project. (2) The mathematical understanding required continually increases. (3) The papers are in danger of being incomprehensible to everyone. Biologists find them much too technical. Mathematicians are likely to find them not written in a way they can easily read. It is time to present the results in a proper way. (4) Grafen has now identified the key

elements for the completely general model, and so will be able to guide the right collaborators in a focussed way to work towards the final answer.

The spirit of the project has three main features. It aims for extreme generality, in order to provide the simplest argument, reflecting the simplicity of the ideas. It aims for a well-defined setting. It also aims to be a meta-model, so that as many existing models of natural selection as possible fall within the domain of the new model, thus allowing a simple principled interpretation of their diverse results. The model should thus apply to all genes, including those newly discovered and sequenced.

The work will be published in journals such as *J. Math. Biol.*, as it is likely to become more rather than less technical, but to influence biologists, papers must appear elsewhere. Three papers take on the important task of communicating the ideas to biologists. ‘The formal Darwinism project: a mid-term report’ (Grafen 2007b) gives a non-technical account. ‘The Simplest Formal Argument for Fitness Optimisation’ (Grafen 2008) gives an elementary mathematical introduction to the project. Grafen (2009) discusses the project non-technically in the context of other recent technical work on inclusive fitness. Another set of papers represent applications of the central core of the project. Grafen (2007a) shows that the model of Killingback et al (2006) is not a new mechanism for evolving altruism, but perfectly ordinary kin selection. Grafen (2007c) shows that kin selection provides an insightful and biologically meaningful analysis of cooperation evolving on graphs (models of Ohtsuki et al 2006 and Ohtsuki and Nowak 2006). Grafen and Archetti (2008) extend the previous paper to a reasonably general treatment of selection in viscous populations. Gardner and Grafen (2009) employ the logic of the formal Darwinism project to provide a test of whether groups can be considered to possess adaptations. These applications will continue, particularly as more general meta-models emerge, but are *not* part of the current proposal, which focuses on the technical core of the project.

### Adjacent work

Taylor (1990, 1996) has already brought together inclusive fitness and a finite number of classes. However, he really offers a formalism, and does not attempt to prove results. He does not compel the view that other models fall within its scope, and makes a number of simplifying assumptions. Taylor and Frank (1996) and Frank (1998) offer advice on how to construct a kin selection model, but this is ‘downwards looking’, towards applications, and does not prove general results. Hammerstein (1996) proposes a ‘streetcar theory’ that, even when it works, guarantees only destinations: a biologist’s concept of fitness also says a lot about selection in progress. Ellner and Rees (2006) have used similarly advanced techniques to the 2006b paper, also to study reproductive value, but in an ecological and demographic, not evolutionary, study. ‘Adaptive Dynamics’ (e.g. Mesz ena et al 2001) makes simplifying assumptions so as to be able to *assume* optimisation, which is then exploited at a higher level, while the formal Darwinism project is precisely about proving links between population genetics and optimisation in the first place. Multi-locus methodology (e.g. Kirkpatrick et al. 2002, Gardner et al 2007) has similar goals of generality about genetic architecture, but does not focus on optimisation. Insisting on retaining dynamic sufficiency means retaining a large formal superstructure for the genetics, and it may be doubted whether this can be managed at the same time as the complexities introduced, for example, by sophisticated phenotypes and linking to optimisation. It is important to monitor and learn from these other projects, but none diminishes the significance of the present application. Houston and McNamara (e.g. 1999) present an excellent set of models for particular applications of a kind that are highly relevant to biologists – an important goal of the formal Darwinism project is to contain as many as possible of these models as special cases. This gives a very concrete and biologically significant ‘target’ for the abstraction. Rousset (2004 and references therein) has recently been providing a very high calibre of analytic result about social selection in subdivided populations. The meta-model of the current proposal may in some sense guarantee that there are dynamically sufficient models such as Rousset’s in a very wide variety of situations, and it will be interesting to classify kinds of conclusions into those that follow from the meta-model and those that really need the more detailed models. Work by Lande and collaborators (Engen et al. 2009a,b) extends understanding of uncertainty in demography and selection in an age-structured population, using a quantitative genetics framework. This may be useful in providing both guidance in the more general setting of the formal Darwinism project, and a test for the meta-model aimed for, as the Banach space of classes permits a representation of age-structure, by using constructed inheritance rules.

## Beneficiaries

Research projects that are based on optimisation ideas in biology will all potentially benefit from this project. Whole organism studies that use functional ideas are strongly linked to optimisation. Sharpening theoretical understanding of optimisation as produced by natural selection will have implications for the background of these studies. The sharpenings to date are particularly relevant to those dealing with relatedness and altruism, and to those dealing with uncertainty and information use. ESS modellers and mathematical population geneticists will benefit from the clarification of what optimisation means and what it entails. Philosophers and historians of science will benefit from having a formal account of the central ideas of Darwin and the later extensions of natural selection.

## References

(Grafen’s are mainly available at <http://users.ox.ac.uk/~grafen/cv/>.)

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